Two particle correlation f^n

Next consider, $G^{(2)}(r\sigma\,;r'\sigma') =$

$$\langle \psi_0 | a^{\dagger}(r,\sigma)\, a^{\dagger}(r',\sigma')\; a(r',\sigma')\; a(r,\sigma) | \psi_0 \rangle$$

Interpretation :

$$a(r,\sigma)|\psi_0\rangle =$$

N-1 particle state with particle removed from r,σ (= Not eigenstate anymore).

So we are calculating density of particles at r',σ' when the particle is removed from r,σ.

$$G^{(2)}(r\sigma; r'\sigma')$$

$$= \langle \psi_0 | a^+(r,\sigma)\, a(r,\sigma) | \psi_0 \rangle$$
$$\langle \psi_0 | a^+(r',\sigma')\, a(r',\sigma') | \psi_0 \rangle$$

$$- \langle \psi_0 | a^+(r\sigma)\, a(r'\sigma') | \psi_0 \rangle$$
$$\langle \psi_0 | a^+(r'\sigma')\, a(r\sigma) | \psi_0 \rangle$$

$$= \frac{\rho^2}{4} - |G^{(1)}(r-r')|^2 \delta_{\sigma\sigma'}$$

$$\Rightarrow G^{(2)} / (\rho^2/4) = 1 - \frac{9\,[\sin(y) - y\cos(y)]^2}{y^6}$$

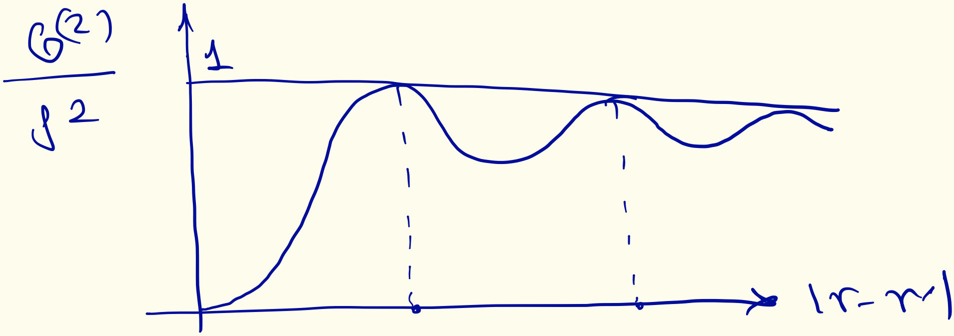
$$y = p_f |(r-r')|$$

$$\lim_{y\to 0} \frac{G^{(2)}}{(\rho^2/4)} = 1 - \lim_{y\to 0} 9 \frac{\left[y - \frac{y^3}{6} - y\left[1 - \frac{y^2}{2}\right] \right]^2}{y^6}$$

$$= 1 - 9\left[\frac{1}{6} - \frac{1}{2}\right]^2$$

$$= 0$$

as expected. since $(a^+)^2 = 0$



$\frac{G^{(2)}}{\rho^2}$
1
|r - r'|

Mean-field Theory

One quick way to capture instabilities is via mean-field theory. In this approach, one assigns a non-zero expectation value to the operator one suspects to be responsible for an instability and finds its value in a self-consistent manner.

Example: Ising model.

See notes from 140 B.

Mean-field Theory of Anti-ferromagnetic instability of Fermions on square lattice.

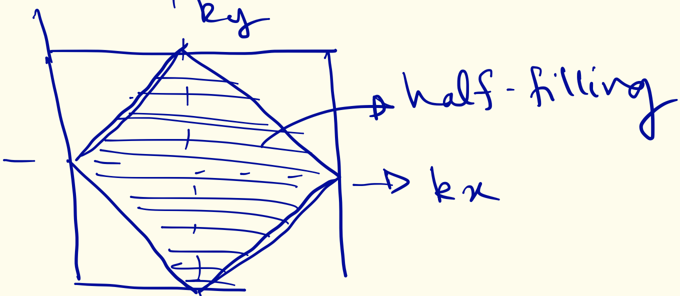
$$H = \sum_k \varepsilon_k \, c^\dagger_{k\sigma} c_{k\sigma} + U \sum_i (\hat{n}_i - 1)^2$$

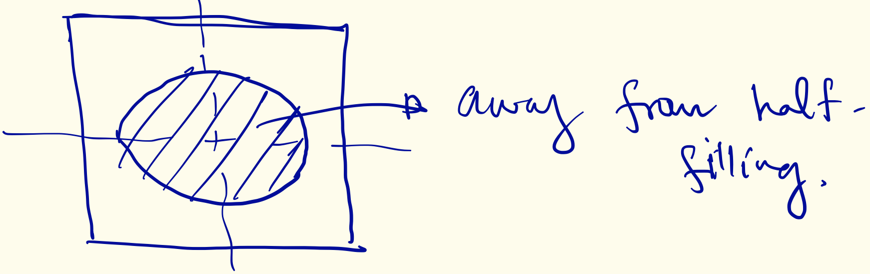
$$n_i = \sum_i c^\dagger_{i\sigma} c_{i\sigma}$$

$$\varepsilon_k = -2t(\cos(k_x) + \cos(k_y)) - \mu$$

μ = chemical potential. When $\mu = 0$

$\Rightarrow$ half-filled fermi surface.





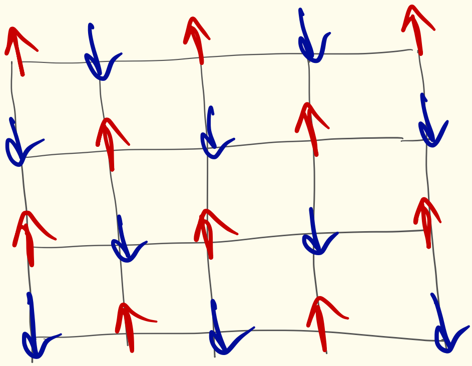
What to expect at half-filling?

First consider the limit $U \gg t$.

As you will show in pset-4, the effective Hamiltonian is:

$$H_{eff} = \frac{4t^2}{U} \sum \vec{S}_i \cdot \vec{S}_j$$

In the ground state of this system spins order antiferromagnetically,



What about the opposite limit $t \gg U$??

Mean-field theory suggests that the system still orders anti-ferromagnetically

At half-filling expect instability for infinitesimal U.

$$U(n_i-1)^2 = -U\left[c^\dagger_i \sigma^z c_i\right]^2 + U$$

$$n_i = 1 \Rightarrow \left[c^\dagger \sigma^z c\right]^2 = 1$$

$$n_i = 0 \Rightarrow \left[c^\dagger \sigma^z c\right]^2 = 0$$

Basic idea of mean-field.

$$(c^\dagger_i \sigma^z c_i)^2 \xrightarrow{\text{Replace}} 2\langle c^\dagger_i \sigma^z c_i\rangle c^\dagger_i \sigma^z c_i - \langle c^\dagger_i \sigma^z c_i\rangle^2$$

Based on physical reasoning $\langle c^\dagger_i \sigma^z c_i\rangle = M(-)^{x+y}$

$$= M(-)^i$$